

Quantifying entanglement in Gaussian states

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Abstract

Entanglement of formation quantifies the entanglement of a state in terms of the entropy of entanglement of the least entangled pure state needed to prepare it. The calculation of this measure is NP-hard and there are only a few cases where we know its analytical expression. In this work we focus on Gaussian states, and for the two-mode case we derive analytical narrow upper and lower bounds, and show that the problem of calculating the actual value for arbitrary states reduces to a trivial optimization process. We apply the measure to a distillation of entanglement experiment and discuss its operational significance. Finally, we propose an extension of this measure for the multipartite case.

Quantifying entanglement is a non-trivial task, since various measures exist with different operational meanings, and most of them lack an analytical expression. Among several entanglement measures, entanglement of formation (EoF) is of significant importance, due to its well-defined physical meaning, i.e., EoF quantifies the entanglement of a state in terms of the entropy of entanglement of the least entangled pure state needed to prepare it [1]. For a given bipartite state $\hat{\sigma} := \sum_i p_i |\psi_i\rangle\langle\psi_i|$, EoF is given by the convex-roof extension of the reduced von Neumann entropy of $|\psi_i\rangle$. For the Gaussian regime EoF is given by $\mathcal{E}(\boldsymbol{\sigma}) := \inf_{\boldsymbol{\sigma}_{p_i}} \{\mathcal{H}_2(\boldsymbol{\sigma}_{p_i}) | \boldsymbol{\sigma} = \boldsymbol{\sigma}_{p_i} + \boldsymbol{\phi}_i\}$, where \mathcal{H}_2 is the entropy of entanglement of a pure state $\boldsymbol{\sigma}_p$ and $\boldsymbol{\phi}$ a positive semi-definite matrix. Using the method of “anti-squeezing” [2, 3] we derive narrow analytical lower and upper bounds to the EoF, i.e., $\mathcal{E}^-(\boldsymbol{\sigma}) \leq \mathcal{E}(\boldsymbol{\sigma}) \leq \mathcal{E}^+(\boldsymbol{\sigma})$. The two bounds become tight for a wide range of states but they can also be considered quite faithful for highly pure states. Since we have an analytical expression for those two bounds we can easily find the exact value of the measure by either maximizing the lower bound or minimizing the upper bound.

We analyse a continuous variable measurement-based entanglement distillation experiment using a variety of entanglement measures. The main conclusion drawn from this analysis [4] is that the most commonly used entanglement measure, i.e., logarithmic negativity, can fail to capture important properties of distillation protocols that are captured by EoF (see also [2, 5]).

In order to extend EoF to multipartite entangled states we need a multipartite version of the von Neumann entropy. Based on Ref. [6] we use a specific form of the α -entanglement entropy defined in the following way: $\mathcal{H}_N := \frac{1}{2} \sum_i^N \mathcal{H}_2(\boldsymbol{\sigma}_{p_i})$. Thus, we define the multipartite EoF (analogously to the bipartite case) as the convex-roof extension of the reduced multipartite von Neumann entropy (as defined above), i.e., $\mathcal{E}_N(\boldsymbol{\sigma}) := \inf_{\boldsymbol{\sigma}_{p_i}} \{\mathcal{H}_N(\boldsymbol{\sigma}_{p_i}) | \boldsymbol{\sigma} = \boldsymbol{\sigma}_{p_i} + \boldsymbol{\phi}_i\}$, that reduces to the bipartite EoF when only 2 of the N modes are entangled with each other.

References

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