Higher-order correlation function $g^{(k)}$ and sub-$k$ photon projection

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Abstract

We analyze the information from higher-order correlation functions $g^{(k)}(0)$ with respect to the projection of the quantum state on the space with less than $k$ photons. As for the special case of $k = 2$, for sufficiently low $g^{(2)}(0)$, this projection obtains an absolute non-zero lower bound, as well as a relative bound for the subspace with at least one photon. Moreover, large-$k$ limits are derived, which allow general lower bounds on these quantities.

The highly nonlinear nature of correlation functions is one of the striking features of quantum physics, which in the last decade has become much more accessible thanks to more refined theory [1] and experiments, the latter being performed in Paderborn [2]. That said, applications of these correlation functions vary drastically between different fields of physics. In semiconductor optics $g^{(2)}(0)$ is measured to identify single-photon sources. Recently, we have analyzed the information gathered from $g^{(2)}(0)$ in this respect [3], showing that this interpretation is independent from the quantum optical notion of subpoissonian or antibunched light. Here, we generalize these ideas to higher orders $k$ [4]. If $g^{(k)}(0)$ falls below the value it attains for the Fock state $|k\rangle$, a nonzero lower bound can be derived for the projection of the density operator on the subspace below $k$ excitations. Likewise, for excluding vacuum from this subspace, one can still obtain a lower bound for the ratio of 1 to $k - 1$ excitations compared to $k$ to infinite excitations, which for $k = 2$ implies a ratio of single to multiple photons. An effective correlation function $\tilde{g}^{(k)}(0)$ is derived that takes vacuum effects into account.

We compare these notions for different known quantum states. Finally we note, see Fig. 1, that for large $k$, these amplitudes approach a steady behavior. We derive expressions for these bounds, which may be used as general lower bounds for the sub-$k$ projection. This work connects the different viewpoints of two fields of physics applying similar concepts for different needs and may further enrich the understanding in both of them.

Figure 1: Lower on bound on the sub-$k$-photon projection of state for different $k$ and large-$k$ limit. From top to bottom $k = 2, 3, 10, \infty$.