Operator ordering sensitivity: measure of nonclassicality of a quantum state

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Abstract

We consider a family of s-parameterized quasidistributions for a single-mode bosonic field and introduce for any state ρ its operator ordering sensitivity as derivative of the second order Renyi entropy of its quasidistribution with respect to s at $s = 0$. We show that this quantity defines a norm on the space of all density operators. On the basis of this norm we introduce a new distance-based measure of nonclassicality for a bosonic field.

The state of a bosonic field is classical if it is a statistical mixture of coherent states, or equivalently, if its Glauber-Sudarshan P-function defines a probability on phase space [1]. Otherwise, it is non-classical. Characterizing and measuring such non-classicality remains an important issue in quantum optics and quantum information theory notably. We introduce a new distance-based measure for non-classicality, and show it outperforms existing such measures in several ways.

The quantum states of a single-mode bosonic field with the annihilation operator a are characterized by quasi-probability distributions W_s which are functions on phase space that depend on an ordering parameter s [2]. The P-function corresponds to $s = 1$ and the Wigner function to $s = 0$. We introduce the ordering sensitivity of the state by

$$S_\mathrm{o} = -\frac{\mathrm{d}}{\mathrm{d}s} \ln ||W_s||^2 = -\frac{1}{2} \frac{\mathrm{Tr}\left([Q,\rho]^2 + [P,\rho]^2\right)}{\mathrm{Tr}\,\rho^2} = |||\tilde{\rho}|||^2, \tag{1}$$

where $||W_s||^2$ is the integral of W_s^2 over phase space, $Q = \frac{1}{\sqrt{2}}(a^\dagger + a)$, $P = \frac{i}{\sqrt{2}}(a^\dagger - a)$, and $\tilde{\rho} = \rho/\sqrt{\mathrm{Tr}(\rho^2)}$. S_o evaluates the sensitivity of the state to operator ordering and measures the oscillations in its Wigner function.

Using the properties of the quasiprobability distributions W_s , we first show that, if the state is classical, then S_o is less than 1. This establishes S_o as a non-classicality witness. Furthermore, pure states are classical iff $S_\mathrm{o} = 1$. We then show that S_o defines a norm on the space of all density operators and hence induces a distance from any state to the set of all classical states $\mathcal{C}$:

$$\mathcal{N}(\rho) = \inf_{\sigma \in \mathcal{C}} |||\tilde{\rho} - \tilde{\sigma}|||. \tag{2}$$

This distance provides a new measure of non-classicality. We show it is easily computable in terms of field quadratures, captures several intuitive features of non-classicality naturally, and detects in many cases non-classicality more efficiently than previously used indicators [3, 4].

Questions arising in quantum information theory drive a continued interest in the exploration of the quantum-classical boundary. There is in this context a need for efficient criteria to determine the strength of the various quantum features of a quantum state. We have concentrated here specifically on the non-classicality question and introduced a new non-classicality criterion that provides an efficient tool for the exploration of the quantum-classical boundary in bosonic systems [5].

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