Quantum coherence and state conversion: theory and experiment

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Abstract

The resource theory of coherence studies the operational value of superpositions in quantum technologies. A key question in this theory concerns the efficiency of manipulation and inter-conversion of the resource. Here we present a full solution of this problem for qubit states by determining the optimal probabilities for mixed state conversions via stochastic incoherent operations. Our theory is supported by experiments with photons, where good correspondence between experiment and theory is found.

In the resource theory of coherence [1], an orthonormal basis of states $\{|i\rangle\}$, usually motivated by physical grounds of being easy to synthesize or store, are considered classical. Any mixture of such states, $\rho = \sum_i p_i |i\rangle\langle i|$, is referred to as "free" and termed *incoherent*, similar to probability distributions on classical states. The free operations are referred to as *incoherent operations* (IO): these are quantum transformations Λ which admit an incoherent Kraus decomposition $\Lambda[\rho] = \sum_i K_i \rho K_i^\dagger$ with incoherent Kraus operators K_i; i.e., $K_i|m\rangle \sim |n\rangle$ for incoherent states $|m\rangle$ and $|n\rangle$. IOs admit a natural interpretation as quantum measurements which cannot create coherence even if postselection is applied on the measurement outcomes. To implement a stochastic IO, we formally postselect a deterministic IO according to the measurement outcomes i.

We present [2, 3] an explicit formula for the maximal probability $P(\rho\to\sigma)$ for converting a single-qubit state ρ into σ via IO: it holds that $P(\rho\to\sigma) = 0$ if $r^2 s_z^2 + \left(1 - r_z^2\right) s^2 > r^2$ and otherwise

$$P(\rho\to\sigma) = \min\left\{\frac{r^2}{(1+|r_z|)\, s^2}\left[1 + \sqrt{1 - \frac{s^2\left(1-r_z^2\right)}{r^2}}\right], 1\right\}, \tag{1}$$

where r_i and s_i are the Bloch coordinates of ρ and σ, and $r = \sqrt{r_x^2 + r_y^2}$. We further report on an optical setup, which realizes IOs with photons (using photon polarization as qubit), and allows to reach optimal conversion probabilities, see Fig. 1 for more details.

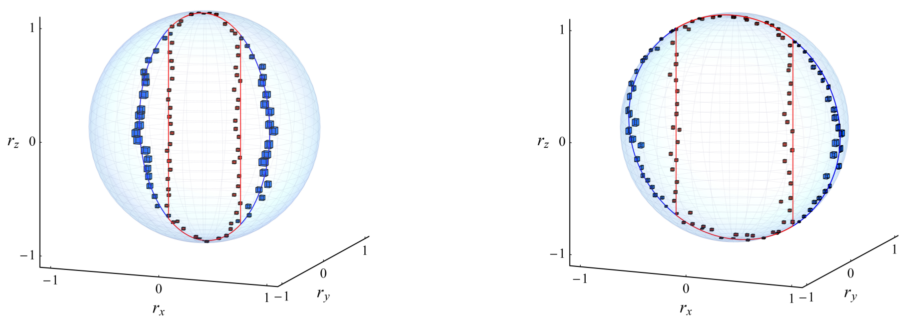

Figure 1: Experimental results for two states with Bloch coordinates $\left(\frac{1}{3}, 0, \frac{5}{6}\right)$ [left figure] and $\left(\frac{\sqrt{11}}{6}, 0, \frac{5}{6}\right)$ [right figure]. The states are prepared with high fidelity up to 0.999. Deterministic and stochastic conversion boundaries in x-z plane are shown in red and blue cubes, respectively, with each side representing the variance $\delta\langle r_i\rangle\,(i = x, y, z)$ derived from Poisson distribution of single photons. Solid lines: theoretical predictions.

[1] A. Streltsov, G. Adesso, and M.B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017).

[2] T. Theurer, A. Streltsov, and M.B. Plenio, arXiv:1804.09467

[3] K.-D. Wu, T. Theurer, G.-Y. Xiang, C.-F. Li, G.-C. Guo, M.B. Plenio, and A. Streltsov, arXiv:1903.01479