CEWQO 2019

Quantum speed of evolution in a finite-temperature bosonic environment

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February 28, 2019

Abstract

In the present work we reconsider from another perspective the standard setting of an input pure one-mode state evolving in contact with a thermal bath described by the quantum optical master equation which is precisely of the Lindblad type. We are here interested in an explicit evaluation of the speed of quantum evolution under this specific master equation which is known to be exactly solvable. We comment on the various approximate treatments of quantum evolution by comparison to the present exact evaluations.

Recently, a lot of work was dedicated to the evaluation of bounds on the speed of quantum evolution for different types of dynamics with both pure and mixed state input: unitary evolution, open and multipartite systems. This is an issue of great interest in some areas of quantum information science, such as quantum computation, quantum metrology, and quantum control as we learn from the recent survey [1]. An insightful derivation of the time-energy uncertainty relation by Mandelstam and Tamm [2] can be considered as the precursor of all research regarding quantum evolution. Thus, for a quantum system governed by a time-independent Hamiltonian $\hat{H}$, the time of evolution between two orthogonal states was found to be bounded by the energy spread $\Delta H = \sqrt{\langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2}$. More recently, however, Margolus and Levitin [3] found that a characteristic time of the system can be related to the average energy $\langle \hat{H} \rangle$ in the initial state. Ar major development was recently achieved with the generalisation of the quantum speed limit to open systems. Taddei et al [4] found an expression in terms of the quantum Fisher information, del Campo et al [5] bounded the rate of change of the relative purity for both Markovian and non-Markovian systems. Deffner and Lutz [6] derived geometric generalizations to open systems of both limits known for unitary evolutions, the Mandelstamm-Tamm bound as well as the Margolus-Levitin one. Geometric quantum speed limits are derived as upper bounds on the rate of change of a geometric measure of distinguishability. For a system initially prepared in a pure state, and then left to evolve under open Markovian dynamics, a practical indicator of its decoherence could be the rate of change of its purity. A treatment based on this quantity was proposed recently in Ref.[7].

Specifically, we here consider a single-mode field in a pure state whose coupling to environment is governed by the quantum optical master equation. We evaluate the characteristic times of quantum evolution according to the treatments in Refs.[5] and [7] and compare them to the exact values resulting from the relative purity and purity respectively, obtained by solving the master equation. Explicit analytical solutions are given for two iconic non-Gaussian states: the Fock state $|M\rangle$ and a definite quantum-mechanical superposition of two coherent states, namely an even coherent state.

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