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Optimal estimation of complex squeezing in phase space

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Abstract

Characterising squeezed states is important for many applications in quantum information processing. We address this through estimation theory and extend previous studies by deriving the fundamental precision bounds of both the magnitude and phase of squeezed Gaussian states. Attaining these bounds through simultaneous measurements is not possible. Instead, separate optimal measurements are demonstrated to be the best strategy to characterise squeezed probe states. For each parameter, we derive the optimal measurement scheme and illustrate how they can be practically implemented using only linear optics.

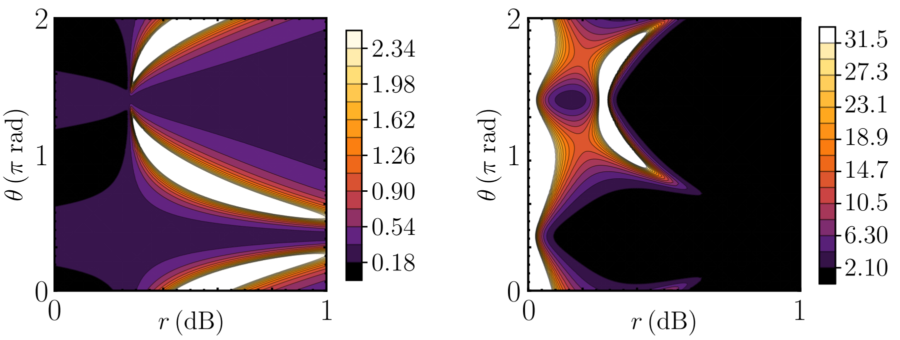
All optical fields fluctuate in both phase and amplitude due to stochastic indeterminacy. This imposes a fundamental shot noise uncertainty to measurements. Squeezed states surpass this precision limit in one quadrature at the expense of a concomitant increased uncertainty to the complementary quadrature. These states have become indispensable in many applications in continuous variable quantum information processing [1] and quantum metrology [2]. To ensure that the amount of squeezing present in a generated squeezed probe is commensurate with its intended application, a complete characterisation of the squeezing magnitude and phase is necessary. In this work, we apply quantum estimation theory to optimally estimate both squeezing parameters.

The single mode squeezer is described by the Hamiltonian [3]

$$\hat{\mathcal{H}} = \frac{ir}{2}\left(e^{-i\vartheta}\hat{a}^2 - e^{i\vartheta}\hat{a}^{\dagger 2}\right). \tag{1}$$

Previous efforts to characterise squeezed states have been limited to estimates of the squeezing magnitude r. We extend this by deriving the generators of translations in both parameters $\boldsymbol{\varphi} = (r, \vartheta)$, and calculating the quantum Cramér-Rao precision bound (QCRB) for estimating the complex squeezing in general Gaussian states. The result is illustrated in figure 1. The QCRB varies a lot over the parameter space, which is not specific to the probe. Notably, there is no value of $\boldsymbol{\varphi}$ that minimises the estimate variances in both parameters simultaneously.

We also address the optimal measurement scheme that attains these fundamental precision bounds. Optimal estimation of both parameters through simultaneous measurements is prohibited, even asymptotically. We show that separate optimal measurements is the best strategy to characterise squeezed probe states, and describe the physical implementation of the estimation scheme using only linear optics. For pure Gaussian states, the optimal measurement scheme is to adaptively apply a 'reverse squeezer' to the squeezed probe followed by intensity measurements and classical post-processing of the data. For mixed Gaussian states, the optimal scheme is a reverse squeezer followed by a displacement of the mode and photon counting.



(a) $\boldsymbol{\mathcal{I}}_r^{-1}$ (b) $\boldsymbol{\mathcal{I}}_\vartheta^{-1}$.

Figure 1: QCRB for the magnitude and direction of complex squeezing as a function of r and ϑ for general Gaussian probe states. The white regions correspond to areas of large variances that are not of interest.

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