

# Sensitivity to initial noise in measurement-induced nonlinear quantum protocols

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## Abstract

We consider iterated quantum protocols with measurement-induced nonlinearity for qubits in the presence of initial noise. We show that in the case where every pure initial state is chaotic, any noisy initial state eventually converges to the completely mixed state. In another protocol, where the pure chaotic states form a fractal, the complex dynamical behavior survives albeit some initial noise, even though the completely mixed state is attractive. Furthermore, the border between convergence regions remains a fractal, and its dimension is constant up to a critical amount of initial noise, where the dimension drops, analogously to a phase transition.

In quantum information theory, the combination of unitary evolution, postselection based on measurement results, and subsequent further manipulation provides a useful tool, e.g. for entanglement distillation. An entangling gate and a subsequent measurement applied on a pair of qubits leads to a nonlinear transformation as originally suggested by Bechmann-Pasquinucci, Huttner and Gisin [1]. The same scheme acting pairwise on an ensemble of identically prepared systems in a pure quantum state may result in a postselected ensemble in a nonlinearly transformed pure state. Iterating such nonlinear quantum state transformations may result in strong dependence on the initial conditions and in complex chaos [2].

We study how initially present noise affects the dynamics in the case of a protocol (the so-called Lattès-type of protocol) where all pure initial states on the Bloch sphere can be considered chaotic. In this case the completely mixed state is an attractive fixed point of the dynamics induced by the protocol. Our numerical simulations strongly indicate that initially mixed states all converge to the completely mixed state. This protocol is an example, where gaining information from measurements and employing it to control an ensemble of quantum systems enables us to create ergodicity, which in turn is destroyed by any initial noise [3].

We consider another protocol, where the pure chaotic states form a fractal structure on the Bloch sphere, the fractal being the border between regions of convergence. We show that the single-qubit dynamics may exhibit sensitive, quasi-chaotic evolution not only for pure initial states but also for mixed states, i.e. the complex dynamical behavior is not destroyed by small initial uncertainty. We show that the appearance of sensitive, complex dynamics associated with a fractal structure in the parameter space of the system has the character of a phase transition. The purity of the initial state plays the role of the control parameter and the dimension of the fractal structure is independent of the purity value after passing the phase transition point. The critical purity coincides with the purity of a repelling fixed point of the dynamics and we show that all the pre-images of states from the close neighborhood of pure chaotic initial states have purity larger than this. Initial states from this set can be considered as quasi-chaotic [4].

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