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Characterizing multipartite entanglement with moments of random correlations

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Abstract

We present a statistical approach for the reference frame independent detection and characterization of multipartite entanglement based on moments of randomly measured correlation functions. We start by showing how the corresponding moments can be evaluated with spherical designs, thus linking methods from both group and entanglement theory. Further on, we illustrated the strengths of the presented strategy in various cases starting with two qubits and followed by more involved multipartite scenarios.

The experimental detection of multipartite entanglement usually requires a number of appropriately chosen local quantum measurements which are aligned with respect to a previously shared common reference frame. The latter, however, can be a challenging prerequisite for photonic free-space quantum communication over distances of several hundreds of kilometers, which is currently in the process of being extended to space involving satellites orbiting the earth [1]. Here, due to the motion, distance and number of involved satellites, the issue of sharing classical reference frames becomes particularly challenging making the development of alternative detection strategies desirable.

One possibility for avoiding the distribution of classical reference frames is to perform a number of local measurements with settings distributed uniformly at random. From the latter one can infer the moments of the corresponding randomly measured correlation functions, which in turn depend on the correlation properties of the considered quantum state ρ. In order to predict the outcome of such an approach we regard a system of N qubits and assume that the local measurement directions $\{\boldsymbol{u}_n\}_{n=1,\ldots,N}$ are chosen uniformly from the Bloch sphere. In this scenario the corresponding moments read:

$$\mathcal{R}^{(t)} = \frac{1}{(4\pi)^N} \int_{S^2} d\boldsymbol{u}_1 \ldots \int_{S^2} d\boldsymbol{u}_N \langle \sigma_{\boldsymbol{u}_1} \otimes \ldots \otimes \sigma_{\boldsymbol{u}_N} \rangle_\rho^t, \tag{1}$$

where $\sigma_{\boldsymbol{u}_i}$ denotes the Pauli measurement corresponding to the direction $\boldsymbol{u}_i$, $d\boldsymbol{u}_i = \sin\theta_i d\theta_i d\phi_i$ is the uniform measure on the Bloch sphere S^2, and t a positive integer.

Following the above framework we were able to show that an improved detection and characterization of multipartite entanglement is possible by combining statistical moments of different order. To do so, we made use of spherical designs which are pseudo-random processes allowing to link methods from group and entanglement theory [2]. We further demonstrated the strengths of our approach in various cases starting with two qubits where it allows for a complete characterization of Bell-diagonal states in terms of the first three non-vanishng moments. Lastly, we turned to more involved multipartite scenarios and showed that a detection and also a characterization of different classes of multipartite entanglement is feasible, as well [3].

In summary, we devised a framework for the reference frame independent detection and characterization of multipartite entanglement based on moments of random correlations. This framework yields advantages compared to previous approaches and has potential applications in the field of photonic free-space quantum communcation.

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[2] J. M. Renes, R. Blume-Kohout, A. J. Scott, C. M. Caves, *Symmetric informationally complete quantum measurements*, Journal of Mathematical Physics **45**, 2171 (2004).

[3] A. Ketterer, N. Wyderka, O. Gühne, *Characterizing multipartite entanglement with moments of random correlations*, arXiv:1808.06558 (accepted in Physical Review Letters).